Średnia harmoniczna $\leq$ Średnia geometryczna $\leq$ Średnia arytmetyczna $\leq$ Średnia kwadratowa

$$\frac{2}{\frac{1}{a}+\frac{1}{b}} \leq \sqrt{ab} \leq \frac{a+b}{2} \leq \sqrt{\frac{a^2+b^2}{2}}$$

$a,b>0$

$$\left.\begin{array}{l} a+\frac{1}{a} \geq 2 \\ \frac{a}{b}+\frac{b}{a} \geq 2 \end{array}\right\} a\cdot b>0 \quad \begin{array}{c}++\\--\end{array}$$

$$\left.\begin{array}{l} a+\frac{1}{a} \leq -2 \\ \frac{a}{b}+\frac{b}{a} \leq -2 \end{array}\right\} a\cdot b<0 \quad \begin{array}{c}+-\\-+\end{array}$$

zad. 6 Udowodnij:

$$\sqrt{\frac{a^2+b^2}{2}} \geq \frac{2}{\frac{1}{a}+\frac{1}{b}}$$

$$\sqrt{\frac{a^2+b^2}{2}} \geq \frac{2ab}{a+b} \quad /^2$$

$$\frac{a^2+b^2}{2} \geq \frac{4a^2b^2}{(a+b)^2}$$

$$\frac{a^2+b^2}{2} - \frac{4a^2b^2}{(a+b)^2} \geq 0$$

$$\frac{(a^2+b^2)(a+b)^2-8a^2b^2}{2(a+b)^2} \geq 0$$

$$\frac{(a^2+b^2)(a^2+2ab+b^2)-8a^2b^2}{2(a+b)^2} \geq 0$$

$$\frac{a^4+2a^3b+a^2b^2+a^2b^2+2ab^3+b^4-8a^2b^2}{2(a+b)^2} \geq 0$$

$$\frac{a^4+2a^3b+2ab^3-6a^2b^2+b^4}{2(a+b)^2} \geq 0$$

$$(a^2-b^2)^2 - 4a^2b^2 + 2a^3b + 2ab^3 \geq 0$$

ważne

$$(a^2-b^2)^2 + 2ab(a^2+b^2-2ab) \geq 0$$

$$(a^2-b^2)^2 + 2ab(a-b)^2 \geq 0$$

II sposób

$$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$$

$$(a-b)^4 = a^4-4a^3b+6a^2b^2-4ab^3+b^4$$

$$a^4+2a^3b+2ab^3-6a^2b^2+b^4 \geq 0$$

do tego momentu tak samo

$$\underbrace{a^4-4a^3b+6a^2b^2-4ab^3+b^4}_{(a-b)^4}+6a^3b-12a^2b^2+6ab^3 \geq 0$$

$$(a-b)^4 + 6ab(a^2-2ab+b^2) \geq 0$$

$$(a-b)^4 + 6ab(a-b)^2 \geq 0$$

III sposób

$a^4 + 2a^3b + 2ab^3 - 6a^2b^2 + b^4 \geq 0 \quad / : a^2b^2$

$\frac{a^2}{b^2} + \frac{b^2}{a^2} + \frac{2b}{a} + \frac{2a}{b} - 6 = \underbrace{\frac{a^2}{b^2} + \frac{b^2}{a^2}}_{\geq 2} + \underbrace{2\left(\frac{b}{a} + \frac{a}{b}\right)}_{\geq 5} - 6 \geq 0$

zad. 26

jeśli $x + y = 1$, to $x^2 + y^2 \geq \frac{1}{2}$

$\Downarrow$

$y = 1 - x$

$x^2 + (1-x)^2 \geq \frac{1}{2}$

$x^2 + 1 - 2x + x^2 \geq \frac{1}{2}$

$2x^2 - 2x - \frac{1}{2} \geq 0$

$4x^2 - 4x - 1 \geq 0$

$(2x-1)^2 \geq 0_{\square}$

zad. 30

$a, b, c, d > 0$

$\sqrt{(a+b)(c+d)} \geq 2\sqrt[4]{abcd} \quad /^2$

$(a+b)(c+d) \geq 4\sqrt{abcd}$

$\frac{(a+b)(c+d)}{4} \geq \sqrt{abcd}$

$\frac{a+b}{2} \cdot \frac{c+d}{2} \geq \sqrt{ab} \cdot \sqrt{cd}$

średnia arytmetyczna dwóch liczb $\geq$ średnia geometryczna dwóch liczb

zad. 36

$\begin{cases} a > b \\ a + 2b < 0 \end{cases} \qquad a(a+b) < 2b^2$

$\left.\begin{cases} b - a < 0 \\ a + 2b < 0 \end{cases}\right\} \Rightarrow (b-a)(a+2b) > 0$

$ab + 2b^2 - a^2 - 2ab > 0 \Rightarrow 2b^2 > a^2 + ab_{\square}$

zad. 38

$a < b < c$

$$\frac{a+2b}{3} < \frac{3b+4c}{7} \quad /\cdot 21$$

$$\Updownarrow$$

$$7a + 14b < 9b + 12c$$

$$\Updownarrow$$

$$7a + 5b < 12c$$

$$L = 7a + 5b < 7b + 5b = 12b < 12c \quad \square$$

zad. 47

$a, b \geqslant 0$ $\qquad$ $a = b$ lub $a + b = 1$

$$\sqrt{a^2 + b} = \sqrt{a + b^2}$$

$$a^2 + b = a + b^2$$

$$a^2 - a = b^2 - b$$

$$a^2 - a - b^2 + b = 0$$

$$(a-b)(a+b) - (a+b) = 0$$

$$(a-b)(a+b-1) = 0$$

$$a = b \quad \vee \quad a+b=1 \quad \square$$